# Test a Point

The solution of an inequality in two variables is a *region* — and the sign does not tell you which side.



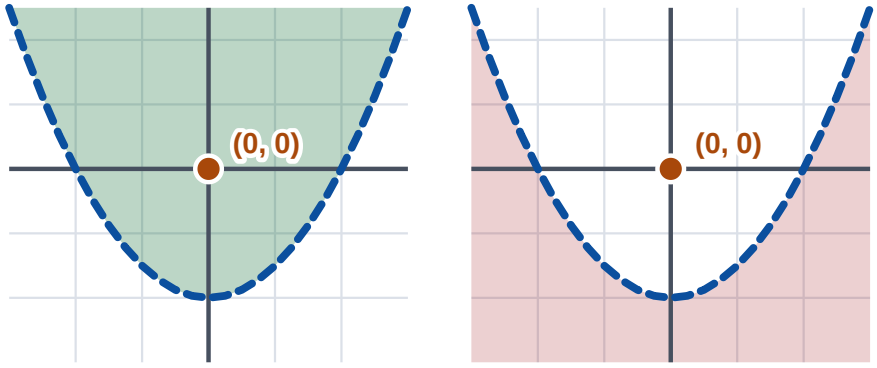
## THE ROUTINE — BOTH STEPS, EVERY TIME

1. **Draw the BOUNDARY — replace the sign with = and graph it.**
2. **TEST A POINT not on the boundary.** TRUE → shade that side. FALSE → shade the other side.

| < > | ≤ ≥ |
|---|---|
| Broken — the boundary points are not solutions | Solid — the boundary points are solutions |

## ONE BOUNDARY, TWO SIDES — THE TEST DECIDES

$y > x^2 - 4$ $\qquad$ $y < x^2 - 4$




$0 > -4$ TRUE → shade this side $\qquad$ $0 < -4$ FALSE → shade the other side

Same curve, same test point, opposite answers. **Nothing but the substitution separates them.**

## THESE TWO ARE THE *SAME* INEQUALITY

$y > x^2 - 4$ at (0, 0): $0 > -4$ TRUE $\quad = \quad$ $x^2 - y < 4$ at (0, 0): $0 < 4$ TRUE

One says **greater** and one says **less**, and it makes no difference at all — moving y across the sign is what flipped it. **The sign depends on how the inequality happens to be written. The region does not.**

## IS A POINT IN, OUT, OR ON?

Substitute. For $y \geq x^2 - 4$:

| POINT | SUBSTITUTION | WHERE | SOLUTION? |
|---|---|---|---|
| (0, 0) | $0 \geq -4$ | IN | yes |
| (2, 0) | $0 \geq 0$ | ON | yes — ≥ includes it |
| (1, −5) | $-5 \geq -3$ | OUT | no |

**ON is not a third region.** It is the edge — and whether it counts depends only on whether the boundary is broken or solid.

## THE ORIGIN IS NOT ALWAYS AVAILABLE

$y > -x$ $\quad y < x^2$ $\quad y \geq 3x$ $\quad y \leq -2x$

**Every one of these boundaries passes through (0, 0). Substituting the origin gives 0 on both sides — a false statement, no warning, and a confidently shaded wrong half.**
**If both sides come out 0, the origin is on the boundary. Pick another point.**

**Don't decide which side. Test a point.**