

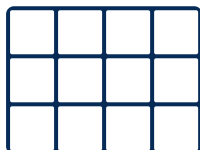
A Part of a Part

Multiplying a fraction by a fraction — shade across, shade down, count the overlap.

HOW TO DO IT — $\frac{2}{3} \times \frac{3}{4}$

1

Draw a grid.
4 columns, 3 rows.



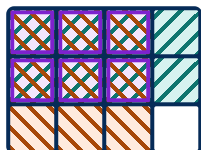
12 cells altogether



first fraction — columns

2

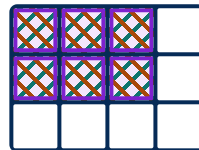
Shade 3 of 4 columns one way,
2 of 3 rows the other.



two directions, one grid

3

Count the cells with both marks.



6 of 12 = $\frac{1}{2}$



the product

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Tops times tops. Bottoms times bottoms. Then simplify.

WHEN IT DOES NOT GET SMALLER

TIMES EXACTLY 1

$$\frac{3}{5} \times 1 = \frac{3}{5}$$

Unchanged. Not smaller.

TIMES MORE THAN 1

$$\frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

Bigger — the multiplier is over 1.

TIMES A WHOLE NUMBER

$$\frac{3}{5} \times 4 = \frac{12}{5}$$

Bigger. That is $2\frac{2}{5}$.

The rule is about the size of the multiplier: under 1 → smaller · exactly 1 → the same · over 1 → bigger.

PRODUCTS YOU SHOULD KNOW BY SIGHT

×	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{15}$
$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{2}{15}$
$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{20}$
$\frac{3}{5}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{3}{25}$

Every answer here is smaller than both fractions that made it — twenty examples on the wall, all year. The table is symmetric, too.

AWKWARD FACTORS

A WHOLE NUMBER — PUT A 1 UNDERNEATH

$$4 = \frac{4}{1} \text{ so } 4 \times \frac{2}{3} = \frac{8}{3}$$

A MIXED NUMBER — MAKE IT IMPROPER FIRST

$$1\frac{1}{2} = \frac{3}{2} \text{ so } \frac{3}{2} \times \frac{2}{3} = 1$$

SIMPLIFY BEFORE YOU MULTIPLY — LESS WORK

$$\frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$$

Cancel 3 with 9, and 4 with 8 — never top with top.

“Of” means multiply. A part of a part is smaller than **both**.